

Steady-state entanglement enhanced by a dissipative ancilla

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We investigate how to enhance entanglement in the steady state of interacting two-level systems. The steady state is reached by spontaneous decay of the individual systems. When we additionally couple them to a dissipative two-level ancilla with variable eigenfrequency and coupling strength, we observe a considerable enhancement effect in the entanglement of this steady state. Moreover, we see that the increased entanglement is directly connected to the selection of certain excited states via the environment disturbing the ancilla. This effect could be used in dissipative state preparation schemes as well as a testbed for decoherence models.

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I. INTRODUCTION

Entanglement is a very peculiar feature and one of the hall marks of quantum mechanics [1, 2]. Its fundamental awkwardness, that surfaces for example in tests of the Bell inequalities [3, 4], has been subject to scientific discussion since the early days of quantum mechanics. Besides these conceptual difficulties in grasping the notion of entanglement, in present-day quantum information theory [5], it is considered as a resource [6]. Many stunning achievements, like quantum teleportation [7], quantum cryptography [8], quantum simulators [9] or quantum computational algorithms [10–12] rely on entangled quantum states more or less heavily.

Usually, the awkwardness of the quantum world stays hidden from our every day observations. The belief is, that environmental decoherence destroys counterintuitive quantum properties, like entanglement. This transition from the quantum to the classical world is one of the central achievements in the theory of decoherence [13–15]. According to this theory, the quantum state of every system in contact with an environment ultimately decays into a classical mixture of states, where the explicit form of these states is governed by the system reservoir interaction.

From this perspective it seems rather conflicting, that environmental decoherence also has been found to be useful [16]. In special systems entanglement can be created by decoherence [17–19] or even quantum computations may be performed dissipatively [20]. The basic idea of engineering dissipative environments to prepare interesting quantum states [21] has been extended to the preparation of entangled quantum states [20, 22, 23]. Nowadays, a plethora of dissipative quantum state preparation schemes has emerged, where the steady state of the system shows the peculiar feature of quantum entanglement, despite or rather just because being subject to the influence of environmental decoherence. In most of them the

environment is engineered by applying active driving of the systems by microwaves or lasers. Such active schemes exist for many different physical systems, like ions in traps [24–26], atoms in cavities [27–33], ensembles of atoms [34–37], directly coupled solid state qubits [38, 39] and solid state qubits in resonators [40–45]. However, there are also passive schemes [38, 39, 46–49], where the steady state of the system can be reached without external driving. In these schemes the interaction between the systems constituents ensures an entangled steady state.

In this manuscript we also examine a passive scheme. We are interested in how steady state entanglement can be enhanced in the simplest bipartite system. Therefore we extend the models of two coupled two-level systems in heat baths, studied for example in Refs. [38, 49], by a third two-level system C , that mediates the coupling to an additional heat bath, see Fig. 1. One might say, that

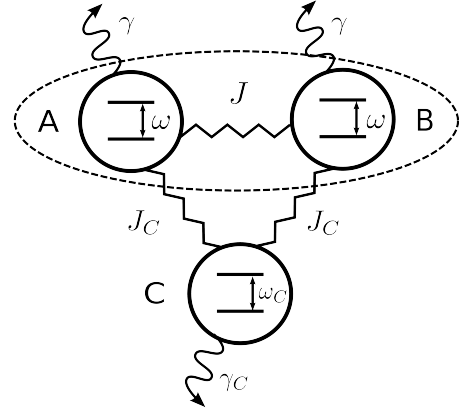


FIG. 1. We are interested in the steady state entanglement between systems A and B and especially how this entanglement depends on the properties of the dissipative ancilla C . The model consists of three two-level systems A, B and C with eigenfrequencies ω and ω_C that are coupled symmetrically, where J and J_C are the coupling strengths. Each of the systems is located in a heat bath, characterized by the decoherence parameters γ and γ_C , respectively.

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this dissipative ancilla, this means system C and its heat bath, forms an interesting, engineered environment with regard to systems A and B . We refer to it as engineered, because we assume to have some control over system C , giving us the chance to manipulate the dissipative ancilla. In this sense controllable two-level systems appear for example in experiments with artificial atoms in superconducting solid state systems [50, 51], where parameters like the eigenfrequency can be tuned. Due to this engineering of an environment we rely on the foundational principles of dissipative state preparation [20–23]. However, in our work we start from an experimentally motivated and very limited setup. In particular, we assume that all coupling operators in the Hamiltonian and the Lindblad operators describing our model of decoherence are fixed. We only vary coupling parameters and eigenfrequencies. In this way, we obtain a practical scheme for preparing entangled states or storing entanglement in presence of a decohering environment.

An additional peculiarity is the treatment of the coupling between our dissipative ancilla C and the systems A and B without any further approximations. Hence we preserve the full quantum dynamics of this interaction, relating our approach to studies of environmentally enhanced entanglement as performed in Ref. [48] for a different decoherence mechanism. Last but not least, the predicted enhancement effect clearly relies on the decoherence model we utilize. This opens up the possibility to sensitively test the phenomenological modeling of decoherence by experimentally checking for such an intricate quantum effect.

Our paper is organized as follows. We start by detailing our model of bipartite system and dissipative ancilla in section II. In section III, we study how to find parameters of the ancilla to enhance the steady state entanglement via dissipation. In section IV we connect this effect to the dissipative preparation of energy eigenstates. This further allows us to understand the optimum of the enhancement. Finally, we arrange our findings and conclude with section V.

II. MODEL

Our model consists of three two-level systems A , B and C , that are coupled symmetrically by a $\sigma_x \sigma_x$ interaction, see Fig. 1. This form of interaction is often found in systems involving artificial atoms, see for example Ref. [38]. All of them are located in thermal baths. In this way we model spontaneous decay. We choose A and B to be equal, i.e. having the same eigenfrequencies $\omega_A = \omega_B = \omega$. Also, these two systems are coupled to the third system with the same interaction strength J_C . This construction allows us to study the entanglement between the two systems A and B and how it depends on the properties of the dissipative ancilla C . The Hamiltonian $H = H_f + H_{int}$ of all three systems combines the

free Hamiltonian

$$H_f = \frac{\hbar\omega}{2} (\sigma_z^A + \sigma_z^B) + \frac{\hbar\omega_C}{2} \sigma_z^C, \quad (1)$$

that describes the free evolutions of the systems A , B and C with their corresponding eigenfrequencies ω and ω_C and the interaction part

$$H_{int} = \hbar J \sigma_x^A \sigma_x^B + \hbar J_C (\sigma_x^A \sigma_x^C + \sigma_x^B \sigma_x^C), \quad (2)$$

which realizes the coupling with strength J between A and B and coupling strength J_C between A and C as well as B and C . In this simplest version of our model, we assume zero temperature heat baths, that are modeled by a coupling of the individual systems to a continuum of harmonic oscillators [52]. After applying the standard Born-Markov approximations, one ends up with a dissipator in Lindblad form [53, 54]

$$\mathcal{L}(\rho) = \sum_{k \in \{A, B, C\}} \gamma_k \left(\sigma_-^k \rho \sigma_+^k - \frac{1}{2} \sigma_+^k \sigma_-^k \rho - \frac{1}{2} \rho \sigma_+^k \sigma_-^k \right), \quad (3)$$

where $\sigma_-^k = |g\rangle\langle e|_k$ is the Lindblad operator that describes spontaneous decay from the excited level $|e\rangle_k$ into the ground state $|g\rangle_k$ of system k and σ_+^k denotes its adjoint. For simplicity, we assume symmetric decoherence parameters $\gamma_A = \gamma_B = \gamma$. The full master equation of our model thus reads

$$\dot{\rho} = L[\rho] = -\frac{i}{\hbar} [H, \rho] + \mathcal{L}(\rho). \quad (4)$$

Before solving it, we introduce the dimensionless time $\tilde{t} = \omega t$, which effectively rescales all eigenfrequencies, coupling strengths and decoherence parameters x by the eigenfrequency ω of the two fundamental systems, i.e. $\tilde{x} = x/\omega$. We drop all tildes and continue with these dimensionless variables.

The steady state solution $\dot{\rho}_{st} = 0$ of Eq. (4) is unique due to the presence of the decay operators σ_- in every subsystem [22, 23]. We find it by solving $L[\rho_{st}] = 0$ numerically [55]. To measure the entanglement between systems A and B , we calculate the Negativity [56, 57]

$$N(\rho_{AB}) = \frac{1}{2} \left(\left\| (\rho_{AB})^{T_B} \right\| - 1 \right) \quad (5)$$

of the reduced steady state density matrix $\rho_{AB} = \text{tr}_C[\rho_{st}]$, where T_B means partially transposed with respect to subsystem B and $\|X\| = \text{tr}[\sqrt{X^\dagger X}]$ is the trace norm.

The model we have chosen is simple, consisting only of three coupled two-level systems in Markovian heat baths. This will allow us to study the influence of tunable parameters like eigenfrequency ω_C or coupling strength J_C of the dissipative ancilla C on the steady state entanglement $N(\rho_{AB})$ between A and B . In addition, such variable eigenfrequencies and coupling strengths immediately lead us to think of experiments with artificial

atoms, realizable in superconducting solid state systems. In those, artificial two-level atoms with tunable eigenfrequencies have already been realized [50, 51]. Depending on the specific implementation, the eigenfrequency can be changed by applying an external voltage or magnetic field. But not only eigenfrequencies are tunable, there are also experiments where the interaction strength between two artificial atoms can be varied, in absolute value as well as in sign [58, 59]. These experiments open up a broad range of in principle accessible parameters, which we want to study in the following.

III. ENHANCEMENT EFFECT

A. Starting parameters

Up to now we have left completely open on how to choose the eigenfrequencies and coupling strengths to obtain an entangled steady state $\rho_{AB} = \text{tr}_C[\rho_{st}]$. Our simple line of guidance will be the case $J_C = 0$ where the two systems A and B are only coupled to each other and their environments, but not to the dissipative ancilla C . In this case the steady state of system A and B alone can be obtained in a simple analytical form [38, 49], for which the Negativity, Eq. (5), reads

$$N(\rho_{AB}) = \max \left[0, \frac{\sqrt{J^2\gamma^2 + 4J^2} - J^2}{4J^2 + 4 + \gamma^2} \right]. \quad (6)$$

We will use this result to motivate our parameters. Remembering that we measure all parameters in our system in units of ω , we pick $\gamma = 10^{-3}$. This seems reasonable, if we look at state of the art experiments [60, 61] where eigenfrequencies of solid state qubits are usually in the range of GHz, and at the same time decoherence parameters are estimated to be in the lower MHz regime. This choice of parameters inserted into Eq. (6) tells us immediately, that for a maximal entanglement of $N = \frac{1}{8}(\sqrt{5} - 1) = 0.155$ we need a coupling strength of $|J| = 0.62$. Hence the steady state entanglement in the AB system should be observable in a strong coupling regime [62–64], where the coupling strength J is almost equal to unity.

This reasoning justifies the initial choice

$$\gamma = 10^{-3}, |J| = 0.62 \quad (7)$$

of our parameters for which the steady state of A and B alone shows the maximal bipartite entanglement of $N = 0.155$. Obviously, if we now switch on the coupling J_C with ancilla C , these parameters do not necessarily describe the point of maximal entanglement between A and B . Nonetheless this is a valuable starting point from which we begin to study the influence of the dissipative ancilla on the entanglement of our bipartite system. To assess our improvements in entanglement, we recall the upper bound of $N = 0.5$, realized by a Bell state [5].

B. Varying eigenfrequency and coupling strength of the dissipative ancilla

Now we want to understand how the bipartite entanglement N , Eq. (5), between A and B varies as a function of the eigenfrequency ω_C and the coupling strength J_C of the ancilla C . At first, we keep the decoherence parameter $\gamma_C = \gamma = 10^{-3}$ fixed. In Fig. 2 we have chosen $J = +0.62$ and observe a merely decaying behavior of the entanglement N with increasing coupling J_C to the dissipative ancilla. This is the somehow expected result: More decoherence in the system reduces steady state entanglement.

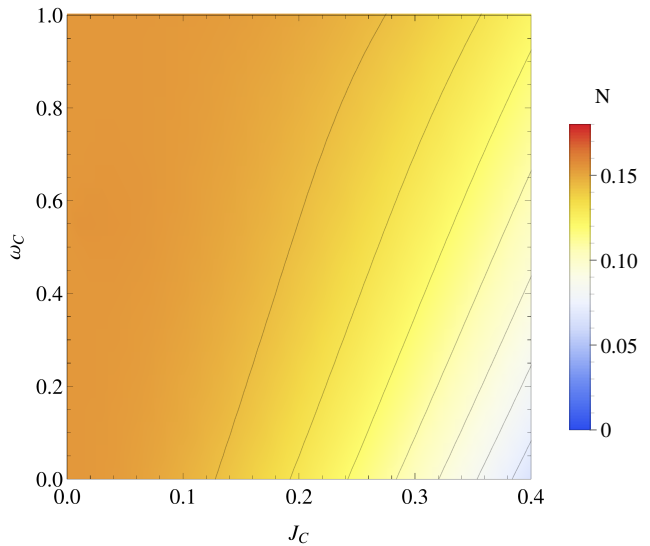


FIG. 2. For $J = +0.62$ and $\gamma_C = \gamma = 10^{-3}$ the steady state entanglement between systems A and B , as measured by the Negativity N , decays with increased absolute value of coupling J_C to the dissipative ancilla C . The entanglement is maximal for $J_C = 0$ and arbitrary ω_C , this means the dissipative ancilla cannot enhance the bipartite entanglement N .

Next we study the case $J = -0.62$. In Fig. 3 we show again the bipartite entanglement N between A and B as a function of the coupling strength J_C and eigenfrequency ω_C of the dissipative ancilla C , keeping the decoherence parameter $\gamma_C = \gamma = 10^{-3}$ fixed. The entanglement does not simply decrease when we increase the interaction strength J_C , but shows a distinctive maximum of $N = 0.180$ (black dot in Fig. 3) for appropriately chosen eigenfrequency $\omega_C = 0.55$ and coupling strength $J_C = 0.01$. Interestingly, here the interaction with the dissipative ancilla C boosts the bipartite entanglement in the steady state of system AB .

The only difference between Figs. 2 and 3, regarding the parameters used to calculate them, is the sign of the coupling strength J . It has been of no importance for the maximal entanglement in the uncoupled case $J_C = 0$, see Eq. (6). However, when we look at the full system ABC , the sign of J is crucial for the behavior of the steady

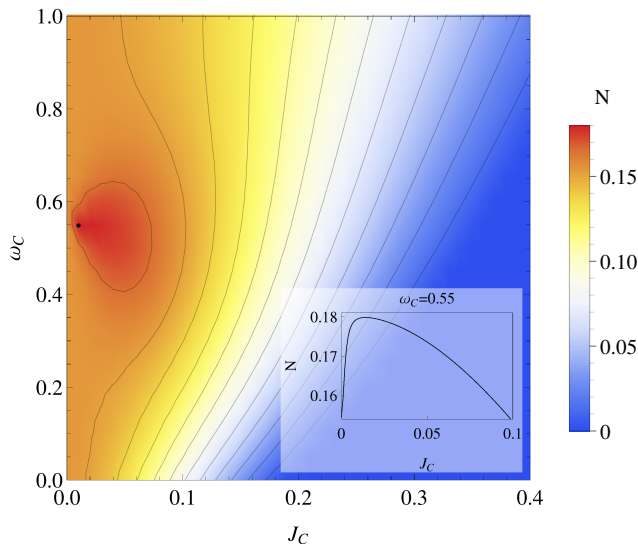


FIG. 3. For $J = -0.62$ and $\gamma_C = \gamma = 10^{-3}$ the entanglement between systems A and B , as measured by the Negativity N , does not simply decrease when we increase the interaction strength J_C with the dissipative ancilla C . There is a distinctive maximum $N = 0.180$ (black dot) of entanglement for an appropriately chosen eigenfrequency $\omega_C = 0.55$ and coupling strength $J_C = 0.01$, also visualized in the inset.

state entanglement. Unfortunately, in this case we have no similarly simple expression for the Negativity as Eq. (6), telling us if the sign of a parameter is of importance or not. A numerical study shows that the sign of the system-ancilla coupling strength J_C is of no importance, but to find local maxima in the steady state entanglement N , it is necessary that the intra-system coupling strength J is negative.

This enhancement effect in bipartite entanglement occurs passively by just adding the dissipative ancilla. We have no active driving elements, like an external laser pumping a specific transition, in our system. Spontaneous decay is present in all three subsystems A , B and C . Yet, as in the case $J_C = 0$ for an uncoupled bipartite system, the steady state ρ_{AB} is still entangled and this bipartite entanglement can even be enhanced. Through the coupling of ancilla C to our bipartite system AB , we obviously increase the space of possible steady states, which can be reached dynamically. Moreover, we retain the full quantum character of this dynamics, as we trace out the dissipative ancilla C without further approximations.

C. Decoherence parameter dependence

In order to fully bring out the importance of the ancilla C coupled to a bath we turn the pike and fix the eigenfrequency $\omega_C = 0.55$ and the coupling strength $J_C = 0.01$ while varying γ_C . Now we investigate the dependence of the bipartite entanglement N on the decay rate γ_C

of our dissipative ancilla, see Fig. 4. Starting from a

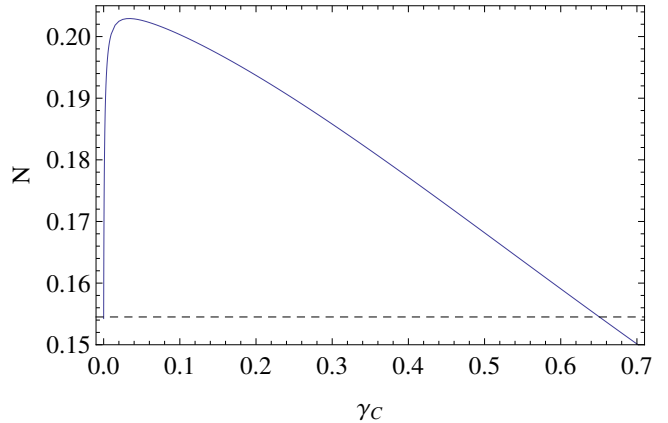


FIG. 4. The value of bipartite entanglement N in dependence of the decoherence rate γ_C . The parameters $\omega_C = 0.55$, $J_C = 0.01$, $J = -0.62$ and $\gamma = 10^{-3}$ are kept constant. Only for values of $\gamma_C > 0.64$ the enhancement effect ceases to exist and the entanglement in the uncoupled system of $N = 0.155$ (black dashed line) is superior.

small γ_C the entanglement increases rapidly to an extremal value of $N = 0.203$ at $\gamma_C = 0.04$ to then decrease again. However, we find the enhanced entanglement over a wide range of dissipation until we reach $\gamma_C > 0.64$, where the enhancement ceases to exist. In this case the maximal entanglement may only be obtained by decoupling the dissipative ancilla. Nevertheless, in the large regime studied here, more decoherence, as measured by γ_C , leads to more entanglement [65] between A and B .

In a realistic scenario, γ_C is not a parameter to be engineered. In contrast to the eigenfrequencies and coupling strengths, that may be tunable, depending on the actual physical realization of the system [50, 51, 58, 59], one cannot simply change the rate of decoherence. Yet, one could think of methods to increase the spontaneous decay of an artificial atom, for example by placing it in a more or less resonant cavity, that is lossy itself. So, at least in principle, it should be possible to influence γ_C to some degree and thereby maximize the entanglement enhancement effect observed here. On the other hand, even if the decoherence parameter γ_C may not be tuned, still Fig. 4 tells us that there is a broad range of possible decoherence parameters γ_C for which the enhancement effect should be observable.

Thus, arguing from a more fundamental point of view, our setup could also be used to put the decoherence model included in the calculation of this quantum effect to a test. In our model all three systems A , B and C are coupled to individual heat baths. Their interaction in presence of environments leads to the entanglement enhancement effect. If this is only an artifact of our model, that does not appear in a real system, this model cannot be used to describe intricate quantum effects of decoherence properly.

IV. ENHANCEMENT AND THE COMPOSITION OF THE STEADY STATE

A. Basic idea

To understand the observed effect of entanglement enhancement further, we study how the entangled steady state is dissipatively prepared [20–23], starting from the initial state

$$\rho(t=0) = |e\rangle\langle e|_A \otimes |g\rangle\langle g|_B \otimes |g\rangle\langle g|_C, \quad (8)$$

where subsystem A is in the excited state $|e\rangle$, while B and the dissipative ancilla C are in their ground states $|g\rangle$, respectively. Actually, the steady state solution ρ_{st} does not depend on the initial state and hence every choice will lead to the same result after more or less rich dynamics. Here we have chosen just one generic example of an initial state to exemplify how the steady state is reached in time.

In fact we will see that the dissipative dynamics selects a specific eigenstate $|E_n\rangle$ of the undamped three-particle interaction, described by the solution of

$$(H_f + H_{int})|E_n\rangle = E_n|E_n\rangle, \quad (9)$$

with Hamiltonians given by Eqs. (1) and (2). In order to trace this selection process in the course of time, we regard the fidelities

$$F_n(t) \equiv \langle E_n | \rho(t) | E_n \rangle \quad (10)$$

of the time evolved state $\rho(t)$, Eq. (4), and the eigenstates $|E_n\rangle$.

The natural expectation would be that a dissipative time evolution finally drives the three-particle system into its ground state $|E_0\rangle$. This is what we basically observe in Fig. 5(a) for the parameter set

$$J = 0.62, \gamma_C = \gamma = 10^{-3}, \omega_C = 0.55, J_C = 0.01, \quad (11)$$

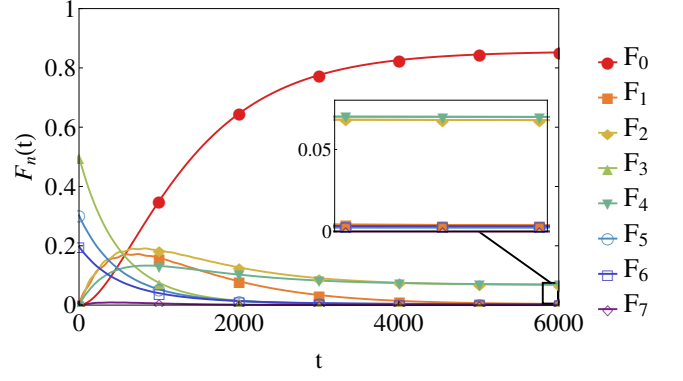
representing the parameter regime also depicted in Fig. 2 of the previous section. Starting from the initial state, Eq. (8), we first observe a transient dynamics, which afterwards merges into the steady state solution. The special fact to be observed in Fig. 5(a) is the mixture in the steady state, as depicted in the inset. We see that besides the expected major contribution of the corresponding ground state $|E_0\rangle$, we still have a small fraction of higher lying eigenstates. Hence we can approximate the steady state density operator by

$$\rho_{st} \approx \sum_{n \in \{0,2,4\}} F_n |E_n\rangle\langle E_n|, \quad (12)$$

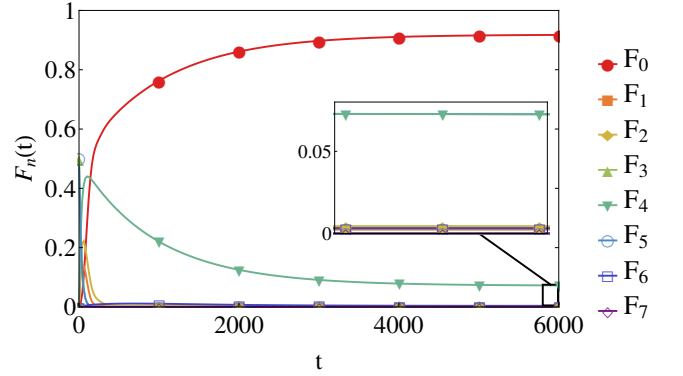
which carries the Negativity $N(\rho_{AB}) = 0.157$ for the AB system in state $\rho_{AB} = \text{tr}_C[\rho_{st}]$ and hence nicely approximates the typical Negativities shown in Fig. 2.

If we now choose the parameter set

$$J = -0.62, \gamma_C = 0.04, \omega_C = 0.55, J_C = 0.01, \gamma = 10^{-3}, \quad (13)$$



(a) $J = +0.62, \gamma_C = 10^{-3}$



(b) $J = -0.62, \gamma_C = 0.04$

FIG. 5. We show the dynamics of the fidelities $F_n(t)$, starting from the initial state in Eq. (8). In particular, we compare the parameter sets from Eqs. (11) and (13) in Figs. 5(a) and 5(b), respectively. We observe in both, that after a transient evolution an equilibrium is reached, where the mixture of the steady state is dominated by the individual ground state $|E_0\rangle$. The insets illustrate, that by changing the sign of J and choosing the optimized decoherence parameter γ_C the mixture is mainly altered by suppressing state $|E_2\rangle$ at the expense of state $|E_0\rangle$. This explains the change in entanglement with respect to subsystem AB .

for which we observed the maximal enhancement effect in the previous section, we recognize a very similar transient dynamics in Fig. 5(b), leading to a strong contribution of the ground state $|E_0\rangle$ [66]. However, the selection of the higher energy eigenstates changes: $|E_2\rangle$ is suppressed at the expense of $|E_0\rangle$, but the state $|E_4\rangle$ survives, so that we arrive at the approximate density operator

$$\rho_{st} \approx \sum_{n \in \{0,4\}} F_n |E_n\rangle\langle E_n|, \quad (14)$$

which indeed leads to the enhanced Negativity $N(\rho_{AB}) = 0.206$, seen in Fig. 4. Obviously, the change in composition of energy eigenstates in the steady state, explains why entanglement is enhanced by dissipatively coupling an ancilla to systems A and B with a suitable choice of parameters. However, the enhancement is still rather

small since the ground state $|E_0\rangle$ dominates this composition. The question then is whether we can even select an eigenstate, like the surviving state $|E_4\rangle$, with a much higher degree of AB entanglement by choosing the right set of parameters. Thus we can try to look at the problem from a fully engineering perspective in the next section.

B. Optimum

In principle, all eigenfrequencies and coupling strengths, and with limitations also the decoherence parameters, are adjustable. In other words, the steady state of the full Liouvillian, Eq. (4), is a function of the parameters J, γ, ω_C, J_C and γ_C . As explained in section III A, with regard to experiment, we fix the reasonable choice $\gamma = 10^{-3}$. Next we constrain the remaining parameters by

$$\begin{aligned} -1 \leq J \leq 0, \quad 0 \leq J_C \leq 1, \\ -1 \leq \omega_C \leq 1, \quad 0 < \gamma_C \leq 1. \end{aligned} \quad (15)$$

These constraints are justified, as we consider similar two-level systems with comparable eigenfrequencies and decoherence parameters. Also, to stay in a physically reasonable parameter regime, the interaction strengths are bounded.

Numerically solving this constrained optimization problem, i.e. finding the steady state solution of Eq. (4) with maximal bipartite entanglement N between systems A and B , yields a maximum of

$$N_{max} = 0.413 \quad (16)$$

for the optimal parameter set

$$J = -0.31, \gamma_C = 0.03, \omega_C = -0.74, J_C = 0.01, \gamma = 10^{-3}. \quad (17)$$

The maximal entanglement $N_{max} = 0.413$ is quite an improvement over the maximal entanglement of $N = 0.155$ in the uncoupled case. At the same time it is again crucial that the ancilla is dissipative, see Fig. 6. The maximum at $\gamma_C = 0.03$ is surrounded by broad sides, where the entanglement is still enhanced. For very small and large values of the decoherence parameter γ_C , the entanglement drops below $N = 0.155$. Hence, we need just the right coupling to a bath for ancilla C to achieve an enhancement effect.

As already mentioned in the previous section, this improvement in entanglement can be explained in more detail, if we once again look at the dynamical selection of eigenstates for $t \rightarrow \infty$. In Fig. 7 we show how the fidelities $F_n(t)$, Eq. (10), evolve in time, again starting from the initial state in Eq. (8). Different than before this time not the ground state $|E_0\rangle$ dominates the mixture, but the excited state $|E_4\rangle$. This is clearly visible from the dynamics, where one can see F_4 rising to its steady state value. As the eigenstate $|E_4\rangle$ possesses bipartite entanglement of $N = 0.499$ in subsystem AB , it is possible

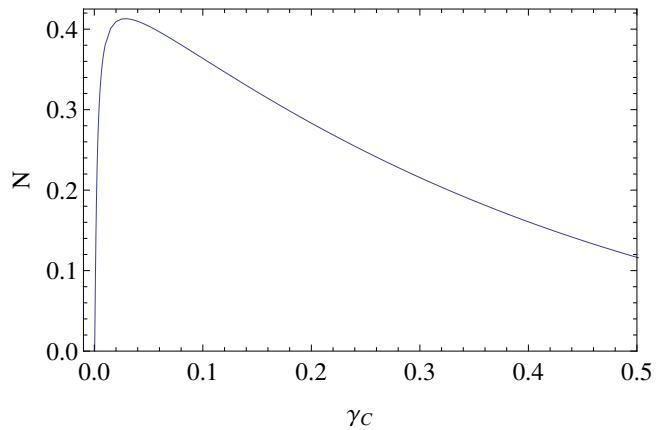


FIG. 6. The value of bipartite entanglement N in dependence of the decoherence rate γ_C . The parameters $\omega_C = -0.74$, $J_C = 0.01$, $J = -0.31$ and $\gamma = 10^{-3}$ are kept constant. As seen before in Fig. 4, the bath of ancilla C is crucial to obtain an enhancement effect.

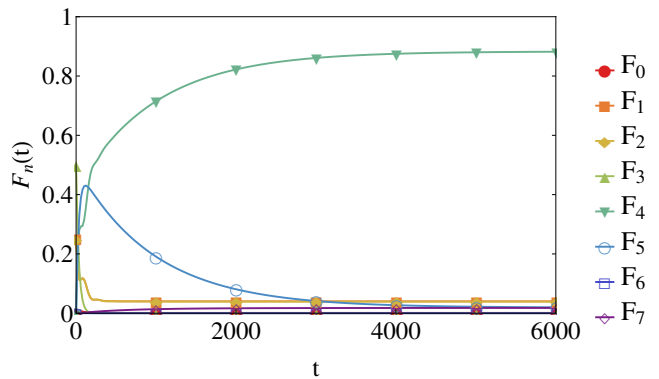


FIG. 7. The dynamics of the fidelities $F_n(t)$, starting from the initial state in Eq. (8) with the parameters from Eq. (17). After a transient evolution an equilibrium is reached where the mixture of the steady state is now dominated by the excited state $|E_4\rangle$. This is in stark contrast to the situation observed in Fig. 5 and explains the significantly different entanglement properties.

to understand why the entanglement is boosted to such a level at this point in parameter space. Here a totally different eigenstate of the Hamiltonian is selected and responsible for the enhancement in bipartite entanglement.

Compared to the maximal possible bipartite entanglement of $N = 0.5$, which is realized by a Bell state, the enhancement that results from coupling systems A and B with an specifically engineered ancilla C is remarkable.

V. CONCLUSION

In this manuscript we have studied how to enhance the steady state entanglement between two two-level systems, by coupling them to a dissipative ancilla. This ancilla has also been chosen to be a two-level system, sub-

ject to the same spontaneous decay noise like the other systems. We have found an enhancement effect in entanglement, depending on the coupling strength, eigenfrequency and decoherence parameter of the ancilla. As we have no active driving elements present, the enhancement occurs passively. Also, the interaction with the dissipative ancilla has been treated without any approximations, i.e. the result contains the full quantum dynamics of the interaction between system and ancilla. We have shown that the enhancement effect is intimately connected to the composition of the steady state, that we expressed in terms of eigenstates of the undamped three-particle system. Coupling a dissipative ancilla has allowed us to alter this composition significantly, enabling the enhancement of entanglement.

The optimal parameters, determined at last, show that a remarkably large enhancement is possible. The associated mixed steady state represents the optimized result of a dissipative state preparation, with respect to the restriction of fixed coupling and Lindblad operators.

Since we have started from an experimentally motivated model, a realization of a related setup might be within reach, establishing the possibility to exploit the observed enhancement effect to engineer entangled states.

Moreover, an experimental approach will shed additional light on our theoretical concepts for the description of decoherence. The more or less phenomenological modeling of spontaneous decay as primary source of decoherence is crucial for the appearance of the enhancement effect. In an experiment, this model would be put to a sensitive test. Beyond that, extensions of the decoherence model, for example regarding finite temperature heat baths or adding additional dephasing noise, are possibilities of further studies.

In addition, due to the model's simplicity and the symmetries involved, at least in some regimes an analytical treatment might be within reach. This could for example be used to study, if there is an even more fundamental mechanism behind the selection of eigenstates explaining the enhancement effect.

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